SuperLU: Sparse Direct Solver and Preconditioner

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Quick installation

- Download site http://crd.lbl.gov/~xiaoye/SuperLU
  - Users' Guide, HTML code documentation
- Gunzip, untar
- Follow README at top level directory
  - Edit make.inc for your platform (compilers, optimizations, libraries, ...) (may move to autoconf in the future)
  - Link with a fast BLAS library
    - The one under CBLAS/ is functional, but not optimized
    - Vendor, GotoBLAS, ATLAS, …

Outline of Tutorial

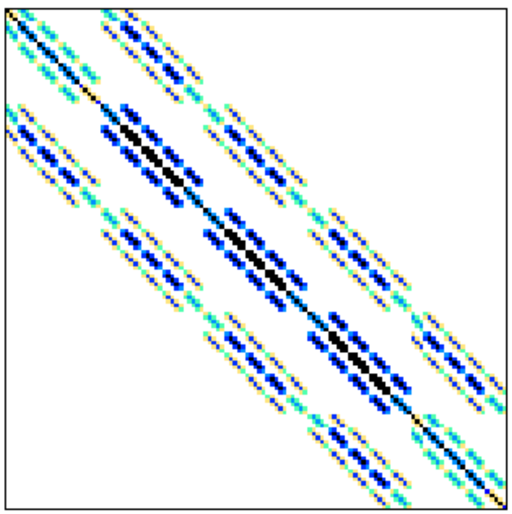
- Functionality
- Sparse matrix data structure, distribution, and user interface
- Background of the algorithms
  - Differences between sequential and parallel solvers
- Examples, Fortran 90 interface

- Hands on exercises

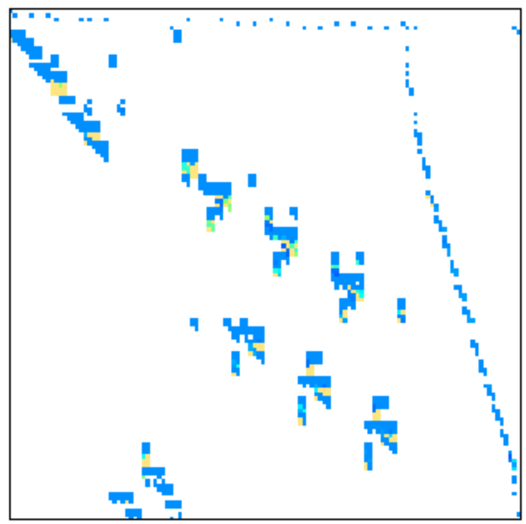
Solve sparse Ax=b : lots of zeros in matrix

- fluid dynamics, structural mechanics, chemical process simulation, circuit simulation, electromagnetic fields, magneto-hydrodynamics, seismic-imaging, economic modeling, optimization, data analysis, statistics, . . .
- Example: A of dimension 10^6, 10~100 nonzeros per row
- Matlab: > spy(A)

Boeing/msc00726 (structural eng.)



Mallya/lhr01 (chemical eng.)



Strategies of sparse linear solvers

- Solving a system of linear equations $Ax = b$
  - Sparse: many zeros in A; worth special treatment
- Iterative methods: (e.g., Krylov, multigrid, …)
  - A is not changed (read-only)
  - Key kernel: sparse matrix-vector multiply
  - Easier to optimize and parallelize
  - Low algorithmic complexity, but may not converge
- Direct methods
  - A is modified (factorized)
  - Harder to optimize and parallelize
  - Numerically robust, but higher algorithmic complexity
- Often use direct method to precondition iterative method
  - Solve an easy system: $M^{-1}Ax = M^{-1}b$

Available direct solvers

- **Survey of different types of factorization codes**

  http://crd.lbl.gov/~xiaoye/SuperLU/SparseDirectSurvey.pdf

  - LL^T (s.p.d.)
  - LDL^T (symmetric indefinite)
  - LU (nonsymmetric)
  - QR (least squares)
  - Sequential, shared-memory (multicore), distributed-memory, out-of-core
    - GPU, FPGA become active.
- **Distributed-memory codes: usually MPI-based**
  - SuperLU_DIST [Li/Demmel/Grigori/Yamazaki]
    - accessible from PETSc, Trilinos, . . .
  - MUMPS, PasTiX, WSMP, . . .

SuperLU Functionality

- **LU decomposition, triangular solution**
- **Incomplete LU (ILU) preconditioner (serial SuperLU 4.0 up)**
- **Transposed system, multiple RHS**
- **Sparsity-preserving ordering**
  - **Minimum degree ordering applied to A^TA or A^T+A [MMD, Liu `85]**
  - **'Nested-dissection' applied to A^TA or A^T+A [(Par)Metis, (PT)-Scotch]**
- **User-controllable pivoting**
  - **Pre-assigned row and/or column permutations**
  - **Partial pivoting with threshold**
- **Equilibration:** $D_r A D_c$
- **Condition number estimation**
- **Iterative refinement**
- **Componentwise error bounds** [Skeel `79, Arioli/Demmel/Duff `89]

Software Status

	SuperLU	SuperLU_MT	SuperLU_DIST
Platform	Serial	SMP, multicore	Distributed memory
Language	C	C + Pthreads or OpenMP	C + MPI + OpenMP + CUDA
Data type	Real/complex, Single/double	Real/complex, Single/double	Real/complex, Double
Data structure	CCS / CRS	CCS / CRS	Distributed CRS

- **Fortran interfaces**
- **SuperLU_MT similar to SuperLU both numerically and in usage**

Usage of SuperLU

- Industry
  - Cray Scientific Libraries
  - FEMLAB
  - HP Mathematical Library
  - IMSL Numerical Library
  - NAG
  - Sun Performance Library
  - Python (NumPy, SciPy)
- Research
  - In FASTMath Tools: Hypre, PETSc, Trilinos, …
  - M3D-C^1, NIMROD (burning plasmas for fusion energys)
  - Omega3P (accelerator design)
  - . . .

Data structure: Compressed Row Storage (CRS)

- **Store nonzeros row by row contiguously**
- **Example: N = 7, NNZ = 19**
- **3 arrays:**
  - **Storage: NNZ reals, NNZ+N+1 integers**

$$\begin{pmatrix} 1 & & & a & & & \\ & 2 & & & b & & \\ c & d & 3 & & & & \\ & e & & 4 & f & & \\ & & & & 5 & & g \\ & & & h & i & 6 & j \\ & & k & & l & & 7 \end{pmatrix}$$

	1	3	5	8	11	13	17	20
nzval	1 a	2 b	c d 3	e 4 f	5 g	h i 6 j	k l 7	
colind	1 4	2 5	1 2 3	2 4 5	5 7	4 5 6 7	3 5 7	

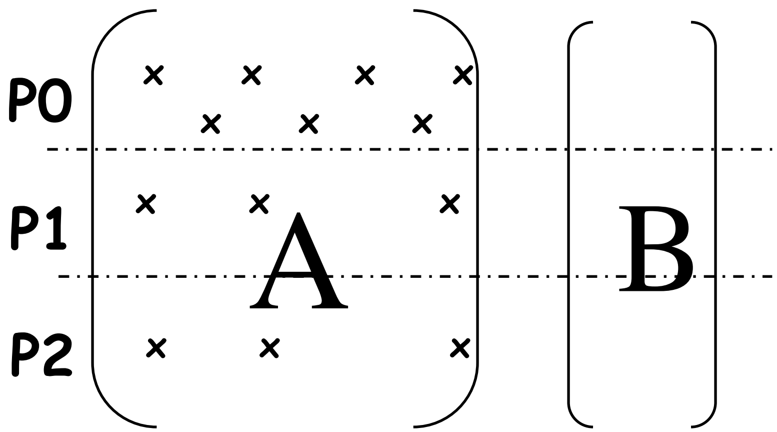
rowptr	1 3 5 8 11 13 17 20

Many other data structures: "Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods", R. Barrett et al.

User interface – distribute input matrices

- **Matrices involved:**
  - **A, B (turned into X) – input, users manipulate them**
  - **L, U – output, users do not need to see them**

- **A (sparse) and B (dense) are distributed by block rows**



Local A stored in
Compressed Row Format

  - **Natural for users, and consistent with other popular packages: e.g. PETSc**

Distributed input interface

- Each process has a structure to store local part of A

  Distributed Compressed Row Storage

```
typedef struct {
    int_t   nnz_loc;  // number of nonzeros in the local submatrix
    int_t   m_loc;    // number of rows local to this processor
    int_t   fst_row;  // global index of the first row
    void    *nzval;   // pointer to array of nonzero values, packed by row
    int_t   *colind;  // pointer to array of column indices of the nonzeros
    int_t   *rowptr;  // pointer to array of beginning of rows in nzval[]and colind[]
}  NRformat_loc;
```

Distributed Compressed Row Storage

A is distributed on 2 processors:

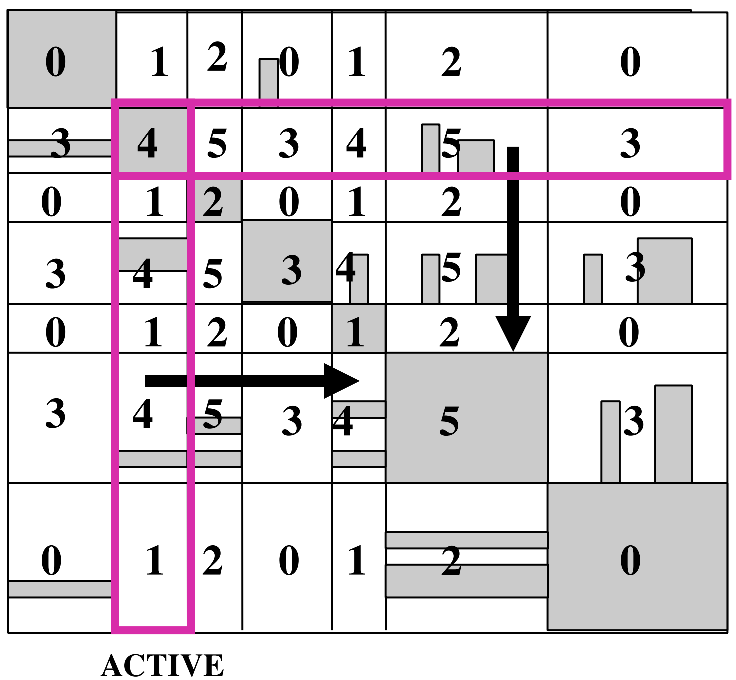
P0	s		u		u
	l	u			
P1		l	p		
				e	u
	l	l			r

- **Processor P0 data structure:**
  - **nnz_loc = 5**
  - **m_loc = 2**
  - **fst_row = 0 // 0-based indexing**
  - **nzval = { s, u, u, l, u }**
  - **colind = { 0, 2, 4, 0, 1 }**
  - **rowptr = { 0, 3, 5 }**
- **Processor P1 data structure:**
  - **nnz_loc = 7**
  - **m_loc = 3**
  - **fst_row = 2 // 0-based indexing**
  - **nzval = { l, p, e, u, l, l, r }**
  - **colind = { 1, 2, 3, 4, 0, 1, 4 }**
  - **rowptr = { 0, 2, 4, 7 }**

Internal : distributed L & U factored matrices

- **2D block cyclic layout**

Matrix



Process mesh

0	1	2
3	4	5

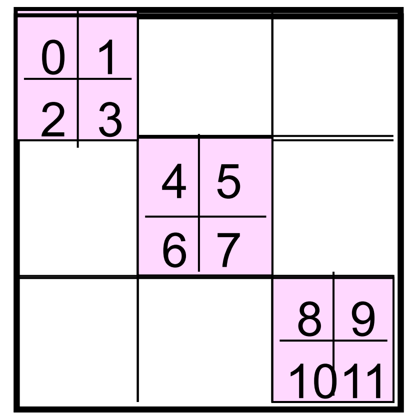
Process grid and MPI communicator

- **Example: Solving a preconditioned linear system**

  $M^{-1}A\,x = M^{-1}\,b$

  $M = \mathrm{diag}(A_{11}, A_{22}, A_{33})$

  → **use SuperLU_DIST for each diagonal block**



- **Create 3 process grids, same logical ranks (0:3), but different physical ranks**
- **Each grid has its own MPI communicator**

Two ways to create a process grid

- **superlu_gridinit(MPI_Comm Bcomm, int nprow,**
  int npcol, gridinfo_t *grid);
  - **Maps the first {nprow, npcol} processes in the MPI communicator Bcomm to SuperLU 2D grid**

- **superlu_gridmap(MPI_Comm Bcomm, int nprow,**
  int npcol, int usermap[], int ldumap, gridinfo_t *grid);
  - **Maps an *arbitrary* set of {nprow, npcol } processes in the MPI communicator Bcomm to SuperLU 2D grid. The ranks of the selected MPI processes are given in usermap[] array.**

    For example:

	0	1	2
0	11	12	13
1	14	15	16

Review of Gaussian Elimination (GE)

- Solving a system of linear equations Ax = b

- First step of GE: (make sure α not too small . . . Otherwise do pivoting)

$$A = \begin{bmatrix} \alpha & \boxed{w^T} \\ \boxed{v} & \boxed{B} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ v/\alpha & I \end{bmatrix} \cdot \begin{bmatrix} \alpha & w^T \\ 0 & C \end{bmatrix}$$

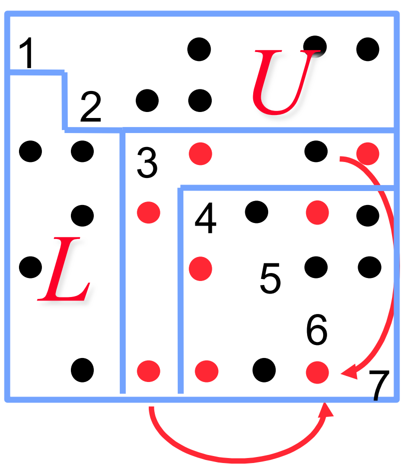
$$C = B - \frac{v \cdot w^T}{\alpha}$$

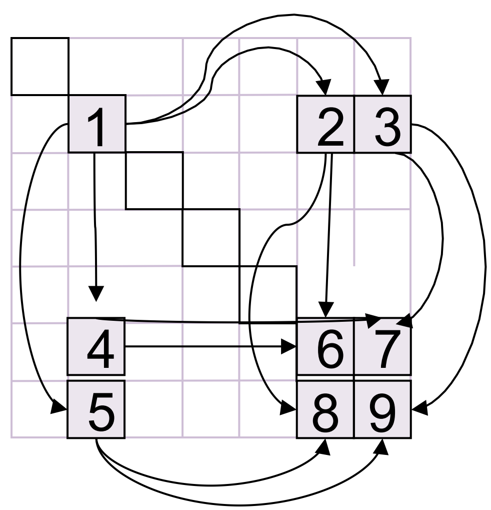
- Repeats GE on C
- Results in {L\U} decomposition (A = LU)
  - L lower triangular with unit diagonal, U upper triangular

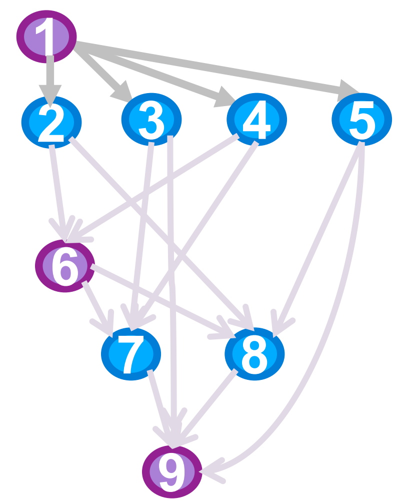
- Then, x is obtained by solving two triangular systems with L and U

Sparse factorization

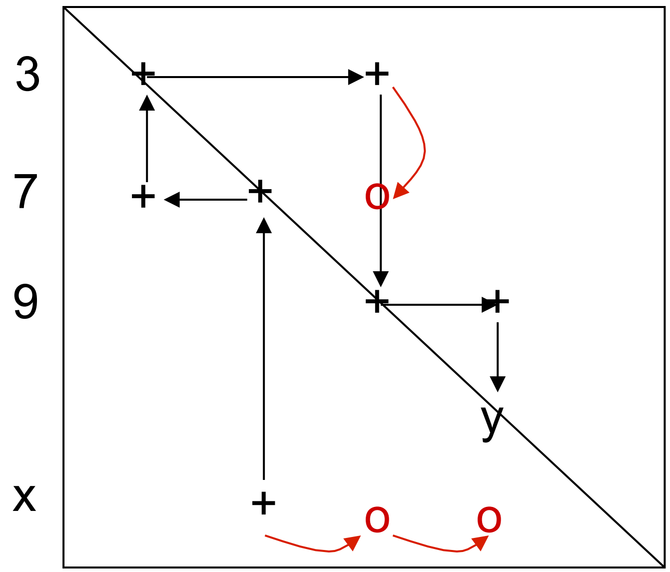
- Store A explicitly … many sparse compressed formats
- "Fill-in" . . . new nonzeros in L & U
  - Typical fill-ratio: 10x for 2D problems, 30-50x for 3D problems
- Graph algorithms: directed/undirected graphs, bipartite graphs, paths, elimination trees, depth-first search, heuristics for NP-hard problems, cliques, graph partitioning, . . .
- Unfriendly to high performance, parallel computing
  - Irregular memory access, indirect addressing, strong task/data dependency







Graph tool: reachable set, fill-path



Edge (x,y) exists in filled graph G^+ due to the path: x → 7 → 3 → 9 → y

- Finding fill-ins ←→ finding transitive closure of G(A)

Algorithmic phases in sparse GE

1. Minimize number of fill-ins, maximize parallelism (~10% time)
   - Sparsity structure of L & U depends on that of A, which can be changed by row/column permutations (vertex re-labeling of the underlying graph)
   - Ordering (combinatorial algorithms; "NP-complete" to find optimum [Yannakis '83]; use heuristics)

2. Predict the fill-in positions in L & U (~10% time)
   - Symbolic factorization (combinatorial algorithms)

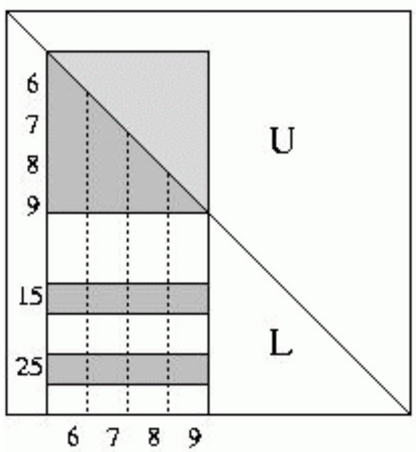
3. Design efficient data structure for storage and quick retrieval of the nonzeros
   - Compressed storage schemes

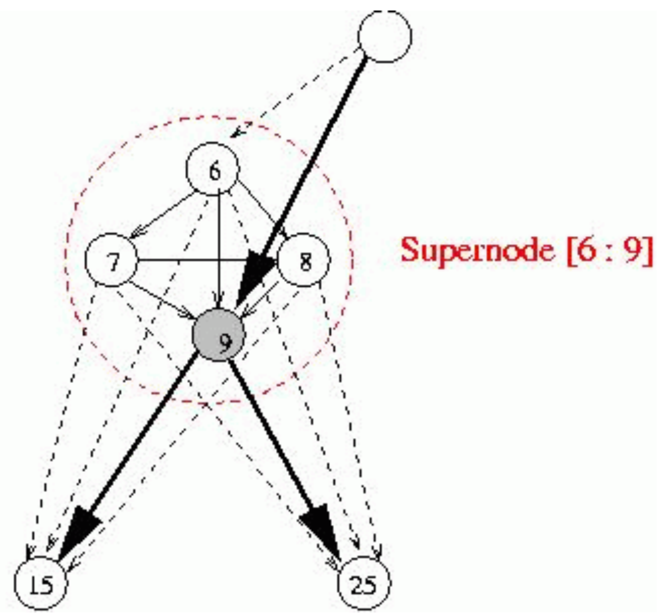
4. Perform factorization and triangular solutions (~80% time)
   - Numerical algorithms (F.P. operations only on nonzeros)
   - Usually dominate the total runtime

- For sparse Cholesky and QR, the steps can be separate;
  for sparse LU with pivoting, steps 2 and 4 my be interleaved.

General Sparse Solver

- Use (blocked) CRS or CCS, and any ordering method
  - Leave room for fill-ins ! (symbolic factorization)
- Exploit “supernode” (dense) structures in the factors
  - Can use Level 3 BLAS
  - Reduce inefficient indirect addressing (scatter/gather)
  - Reduce graph traversal time using a coarser graph

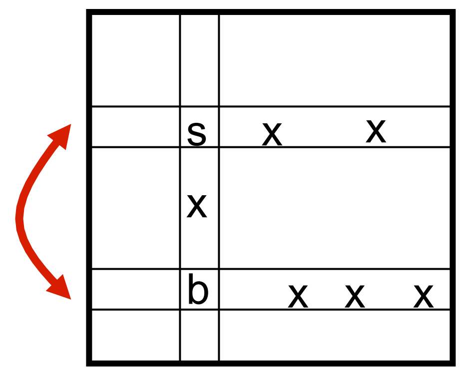




Numerical Pivoting

- Goal of pivoting is to control element growth in L & U for stability
  - For sparse factorizations, often relax the pivoting rule to trade with better sparsity and parallelism (e.g., threshold pivoting, static pivoting , . . .)

- Partial pivoting used in sequential SuperLU and SuperLU_MT (GEPP)
  - Can force diagonal pivoting (controlled by diagonal threshold)
  - Hard to implement scalably for sparse factorization



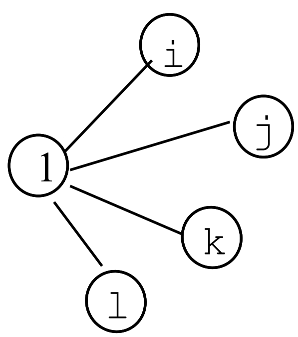
- Static pivoting used in SuperLU_DIST (GESP)
  - Before factor, scale and permute A to maximize diagonal: $P_r D_r A D_c = A'$
  - During factor $A' = LU$, replace tiny pivots by $\sqrt{\varepsilon}\|A\|$, without changing data structures for L & U
  - If needed, use a few steps of iterative refinement after the first solution

  ➔ quite stable in practice

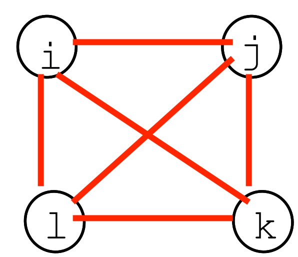
Ordering : Minimum Degree

Local greedy: minimize upper bound on fill-in

$$
\begin{array}{c|ccccc|}
 & & i & j & k & l \\
\hline
1 & x & x & x & x & x \\
i & x & & & & \\
j & x & & & & \\
k & x & & & & \\
l & x & & & &
\end{array}
\xrightarrow{\text{Eliminate 1}}
\begin{array}{c|c|cccc|}
 & & i & j & k & l \\
\hline
1 & x & x & x & x & x \\
\hline
i & x & \bullet & \bullet & \bullet & \bullet \\
j & x & \bullet & \bullet & \bullet & \bullet \\
k & x & \bullet & \bullet & \bullet & \bullet \\
l & x & \bullet & \bullet & \bullet & \bullet
\end{array}
$$

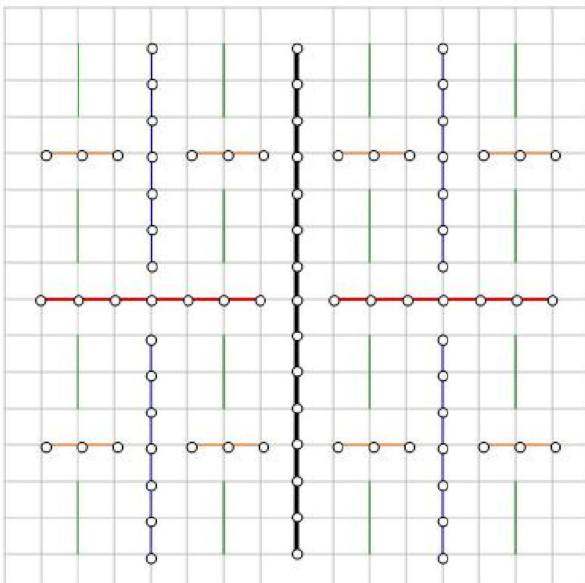


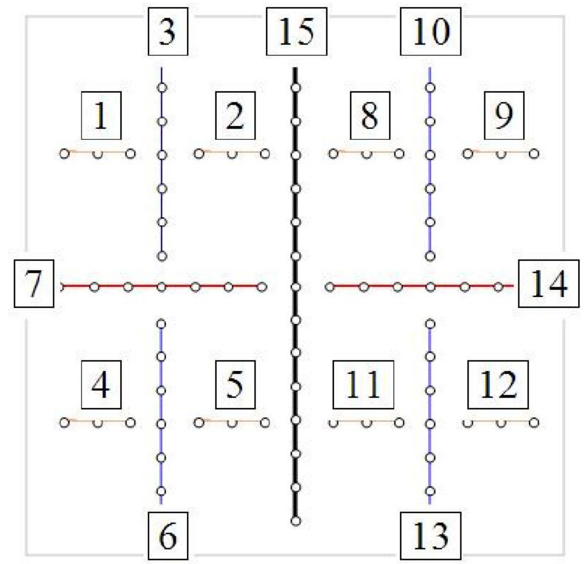
Eliminate 1 $\longrightarrow$



Ordering : Nested Dissection

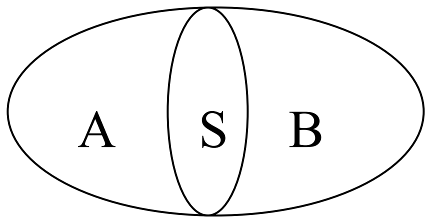
- Model problem: discretized system Ax = b from certain PDEs, e.g., 5-point stencil on n x n grid, $N = n^2$
  - Factorization flops: $O(n^3) = O(N^{3/2})$
- Theorem: ND ordering gives optimal complexity in exact arithmetic [George '73, Hoffman/Martin/Rose]





ND Ordering

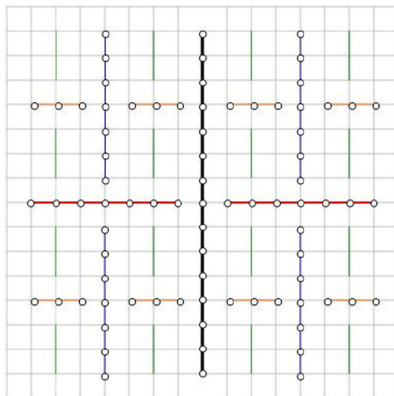
- **Generalized nested dissection** [Lipton/Rose/Tarjan ’79]
  - **Global graph partitioning: top-down, divide-and-conqure**
  - **Best for largest problems**
  - **Parallel codes available: ParMetis, PT-Scotch**
  - **First level**



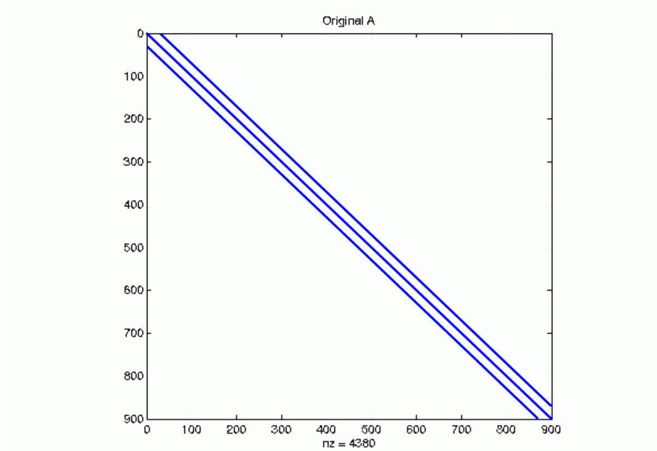
$$\longrightarrow \begin{bmatrix} A & 0 & x \\ 0 & B & x \\ x & x & S \end{bmatrix}$$

  - **Recurse on A and B**
- **Goal: find the smallest possible separator S at each level**
  - **Multilevel schemes:**
    - Chaco [Hendrickson/Leland `94], Metis [Karypis/Kumar `95]
  - **Spectral bisection [Simon et al. `90-`95]**
  - **Geometric and spectral bisection [Chan/Gilbert/Teng `94]**

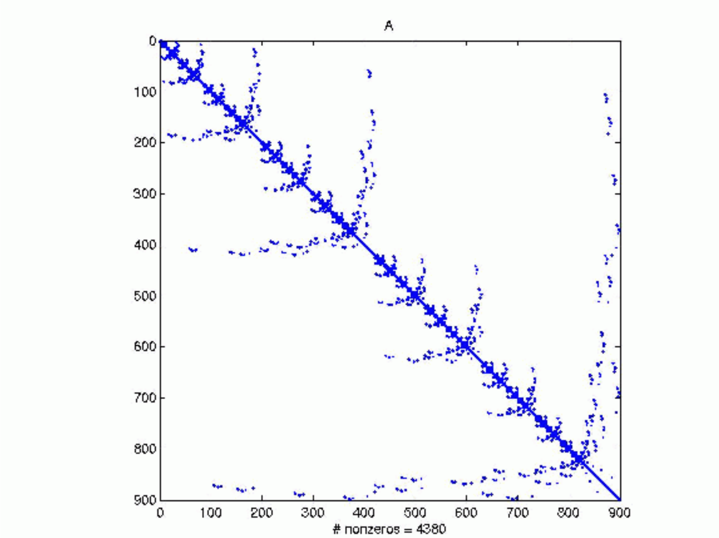
ND Ordering



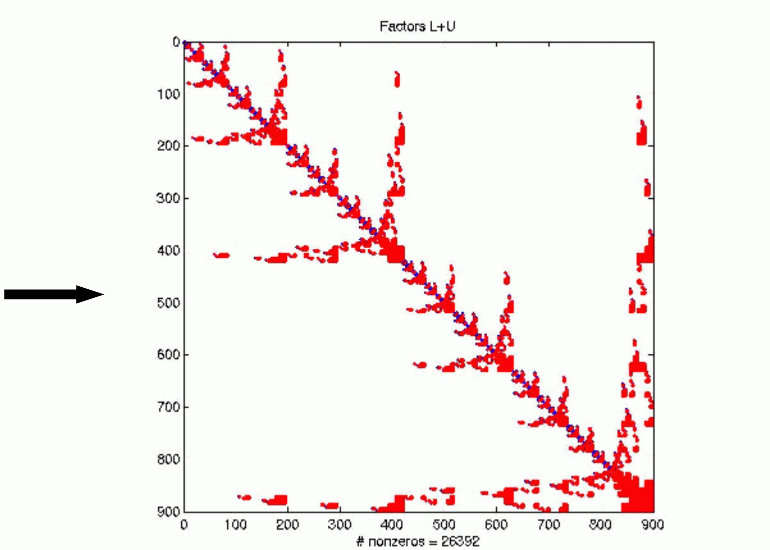
2D mesh



A, with row-wise ordering



A, with ND ordering



L &U factors

Ordering for LU (unsymmetric)

- Can use a symmetric ordering on a symmetrized matrix
  - Case of partial pivoting (serial SuperLU, SuperLU_MT):

    Use ordering based on A^T*A
  - Case of static pivoting (SuperLU_DIST):

    Use ordering based on A^T+A

- Can find better ordering based solely on A, without symmetrization
  - Diagonal Markowitz [Amestoy/Li/Ng `06]
    - Similar to minimum degree, but without symmetrization
  - Hypergraph partition [Boman, Grigori, et al. `08]
    - Similar to ND on A^TA, but no need to compute A^TA

Ordering Interface in SuperLU

- Library contains the following routines:
  - Ordering algorithms: MMD [J. Liu], COLAMD [T. Davis]
  - Utility routines: form A^T+A , A^TA

- Users may input any other permutation vector (e.g., using Metis, Chaco, etc.)

```
. . .
set_default_options_dist ( &options );
options.ColPerm = MY_PERMC;    // modify default option
ScalePermstructInit ( m, n, &ScalePermstruct );
METIS (  . . . , &ScalePermstruct.perm_c );
. . .
pdgssvx ( &options, . . . , &ScalePermstruct, . . . );
. . .
```

Symbolic Factorization

- **Cholesky** [George/Liu `81 book]
  - **Use elimination graph of L and its transitive reduction (elimination tree)**
  - **Complexity linear in output: O(nnz(L))**

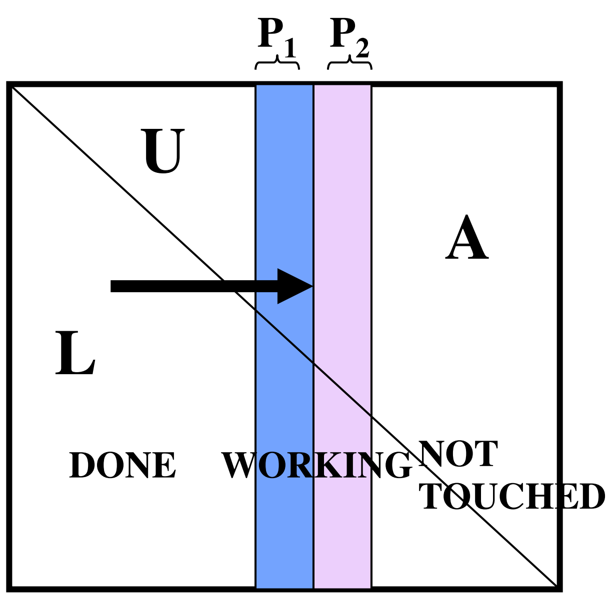
- **LU**
  - **Use elimination graphs of L & U and their transitive reductions (elimination DAGs)** [Tarjan/Rose `78, Gilbert/Liu `93, Gilbert `94]
  - **Improved by symmetric structure pruning** [Eisenstat/Liu `92]
  - **Improved by supernodes**
  - **Complexity greater than nnz(L+U), but much smaller than flops(LU)**

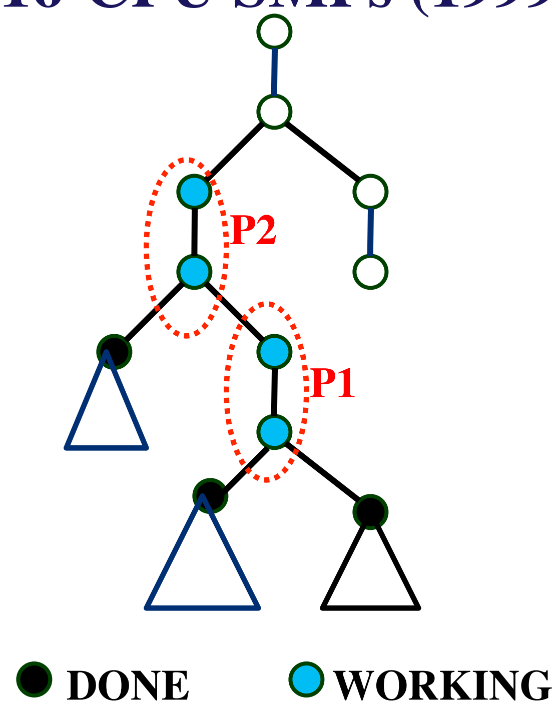
Numerical Factorization

- Sequential SuperLU
  - Enhance data reuse in memory hierarchy by calling Level 3 BLAS on the supernodes
- SuperLU_MT
  - Exploit both coarse and fine grain parallelism
  - Employ dynamic scheduling to minimize parallel runtime
- SuperLU_DIST
  - Enhance scalability by static pivoting and 2D matrix distribution

SuperLU_MT [Li/Demmel/Gilbert]

- Pthread or OpenMP
- Left-looking – relatively more READs than WRITEs
- Use shared task queue to schedule ready columns in the elimination tree (bottom up)
- Over 12x speedup on conventional 16-CPU SMPs (1999)

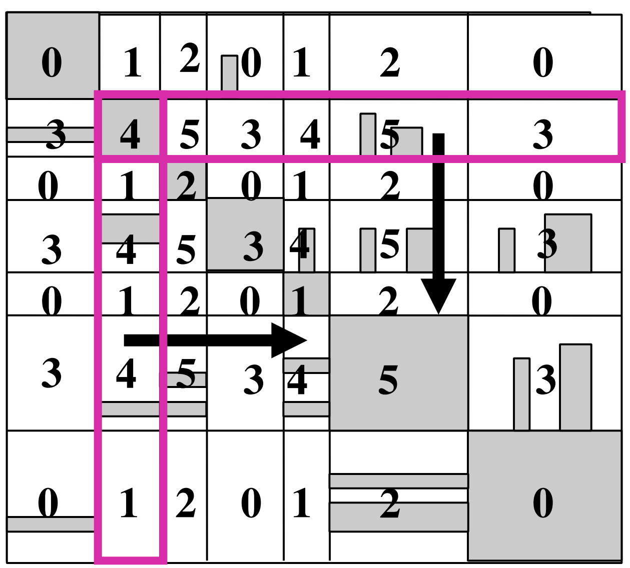




SuperLU_DIST [Li/Demmel/Grigori/Yamazaki]

- MPI
- Right-looking – relatively more WRITEs than READs
- 2D block cyclic layout
- Look-ahead to overlap comm. & comp.
- Scales to 1000s processors

Matrix



ACTIVE

Process mesh

0	1	2
3	4	5

Multicore platforms

- **Intel Clovertown:**
  - 2.33 GHz Xeon, 9.3 Gflops/core
  - 2 sockets x 4 cores/socket
  - L2 cache: 4 MB/2 cores

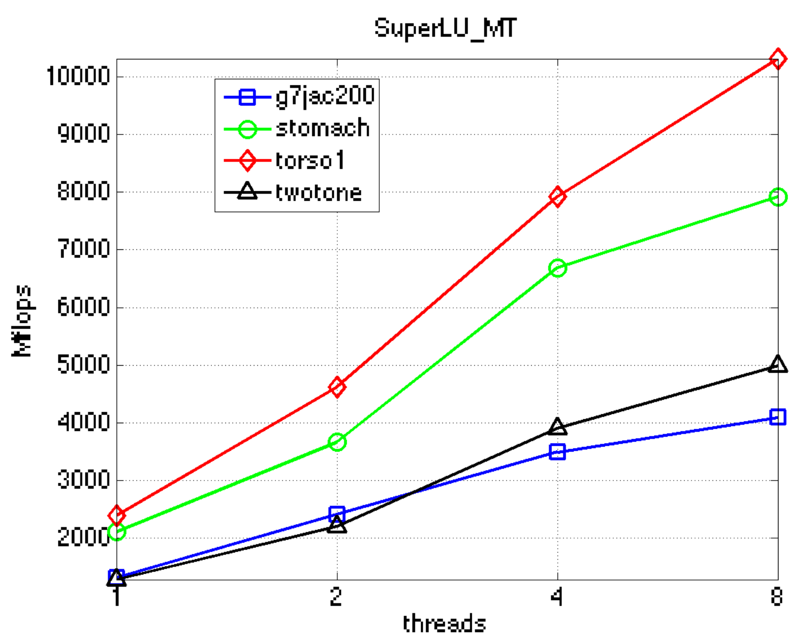
- **Sun VictoriaFalls:**
  - 1.4 GHz UltraSparc T2, 1.4 Gflops/core
  - 2 sockets x 8 cores/socket x 8 hardware threads/core
  - L2 cache shared: 4 MB

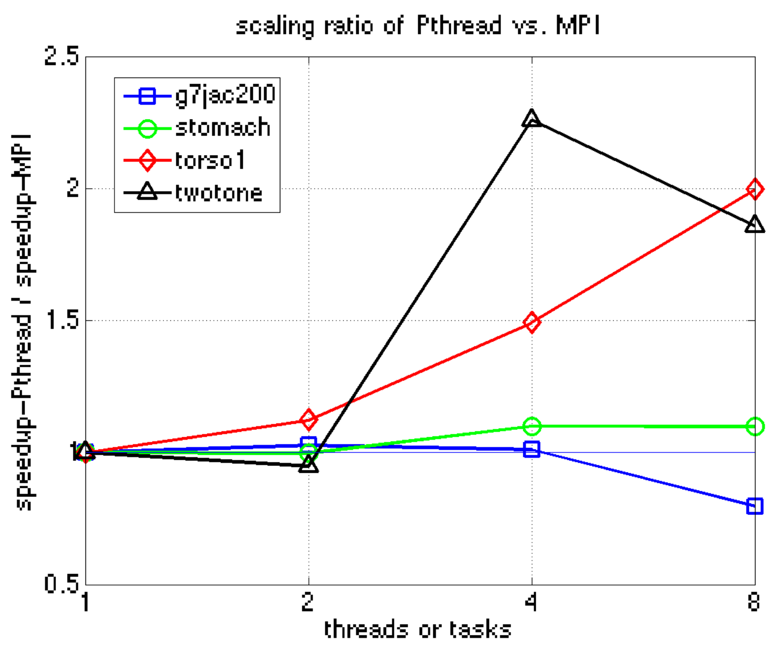
Benchmark matrices

	apps	dim	nnz(A)	SLU_MT Fill	SLU_DIST Fill	Avg. S-node
g7jac200	Economic model	59,310	0.7 M	33.7 M	33.7 M	1.9
stomach	3D finite diff.	213,360	3.0 M	136.8 M	137.4 M	4.0
torso3	3D finite diff.	259,156	4.4 M	784.7 M	785.0 M	3.1
twotone	Nonlinear analog circuit	120,750	1.2 M	11.4 M	11.4 M	2.3

Intel Clovertown

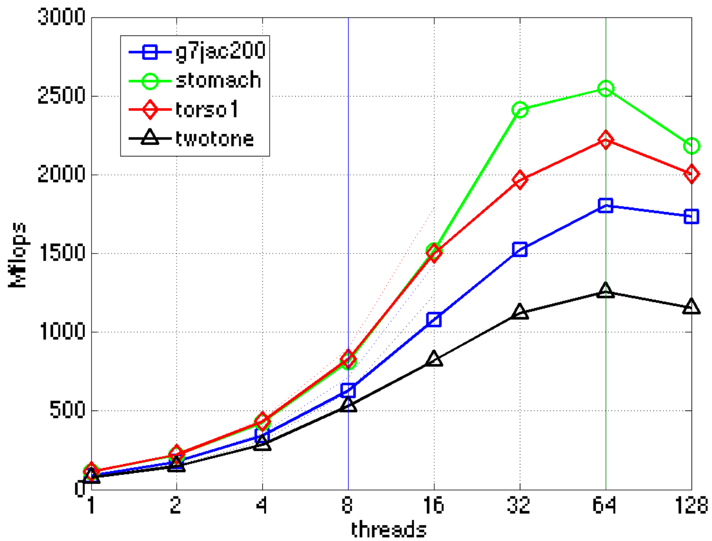
- ❖Maximum speedup 4.3, smaller than conventional SMP
- ❖Pthreads scale better
- ❖Question: tools to analyze resource contention?



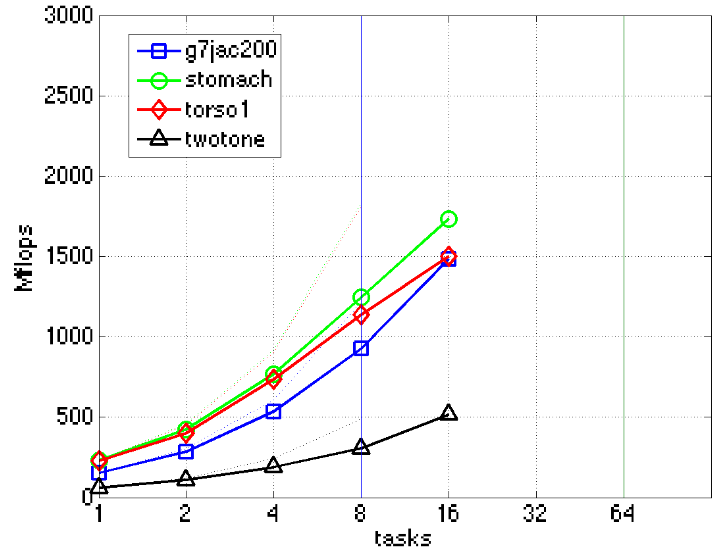


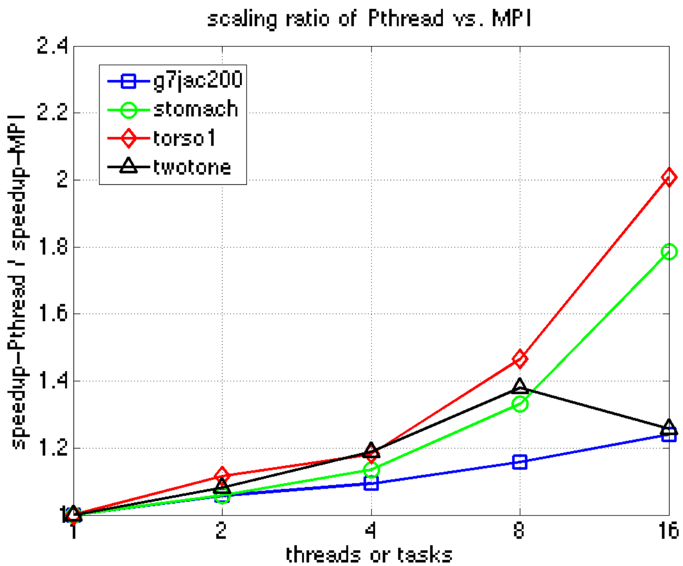
SunVictoriaFalls – multicore + multithread

SuperLU_MT



SuperLU_DIST





- Maximum speedup 20
- Pthreads more robust, scale better
- MPICH crashes with large #tasks, mismatch between coarse and fine grain models

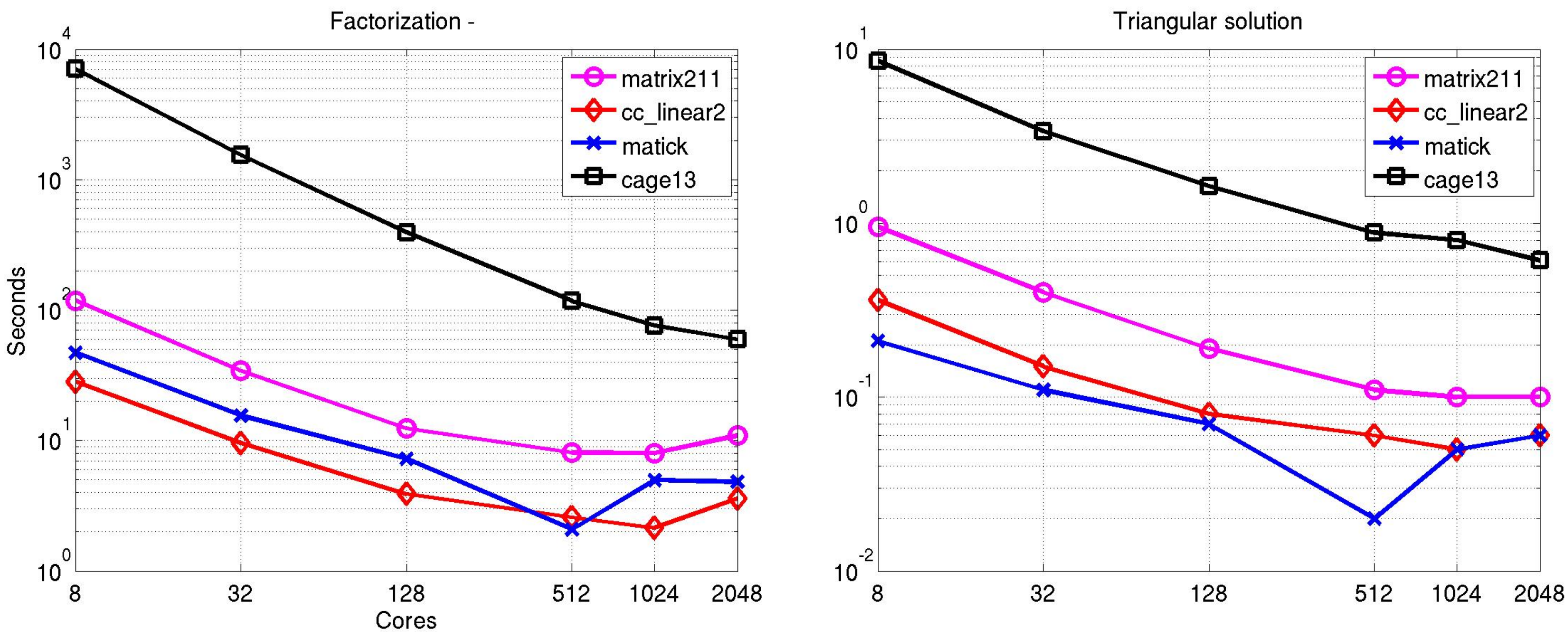
Performance of larger matrices

Name	Application	Data type	N	|A| / N Sparsity	|L\U| (10^6)	Fill-ratio
matrix211	Fusion, MHD eqns (M3D-C1)	Real	801,378	161	1276.0	9.9
cc_linear2	Fusion, MHD eqns (NIMROD)	Complex	259,203	109	199.7	7.1
matick	Circuit sim. MNA method (IBM)	Complex	16,019	4005	64.3	1.0
cage13	DNA electrophoresis	Real	445,315	17	4550.9	608.5

- **Sparsity ordering: MeTis applied to structure of A' +A**

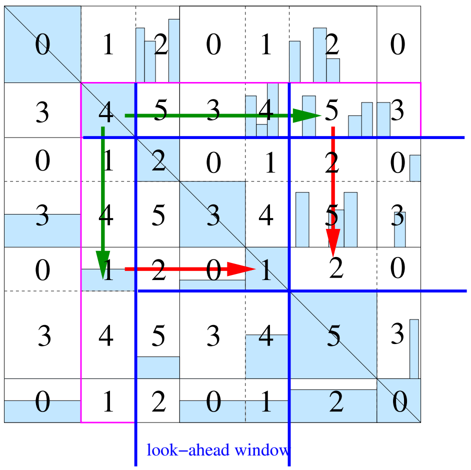
Strong scaling (fixed size): Cray XE6 (hopper@nersc)

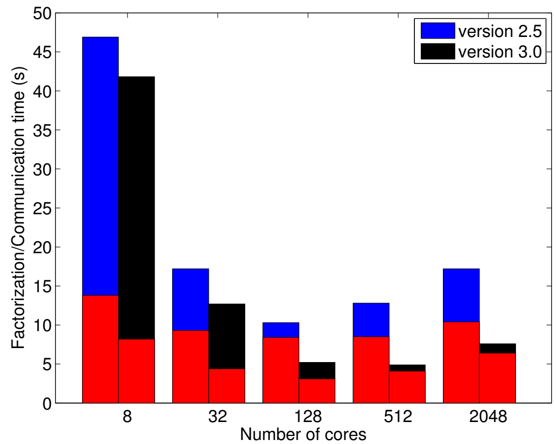
- 2 x 12-core AMD 'MagnyCours' per node, 2.1 GHz processor



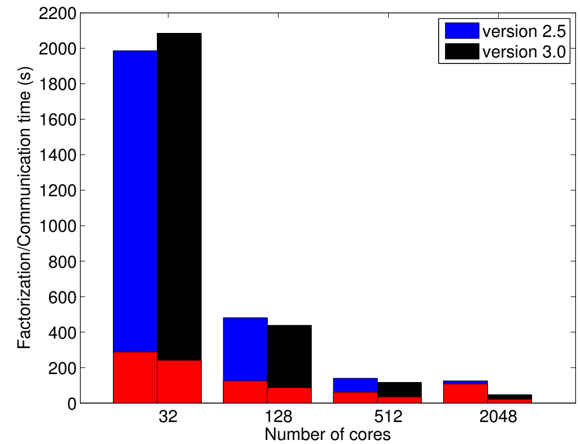
❖ Up to 1.4 Tflops factorization rate

SuperLU_DIST 3.0: better DAG scheduling





Accelerator, n=2.7M, fill-ratio=12



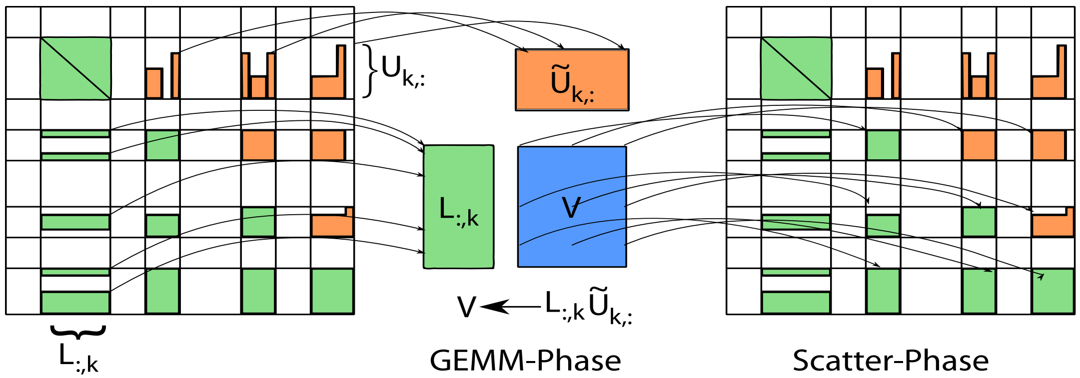
DNA, n = 445K, fill-ratio= 609

- Implemented new static scheduling and flexible look-ahead algorithms that shortened the length of the critical path.
- Idle time was significantly reduced (speedup up to 2.6x)
- To further improve performance:
  - more sophisticated scheduling schemes
  - hybrid programming paradigms

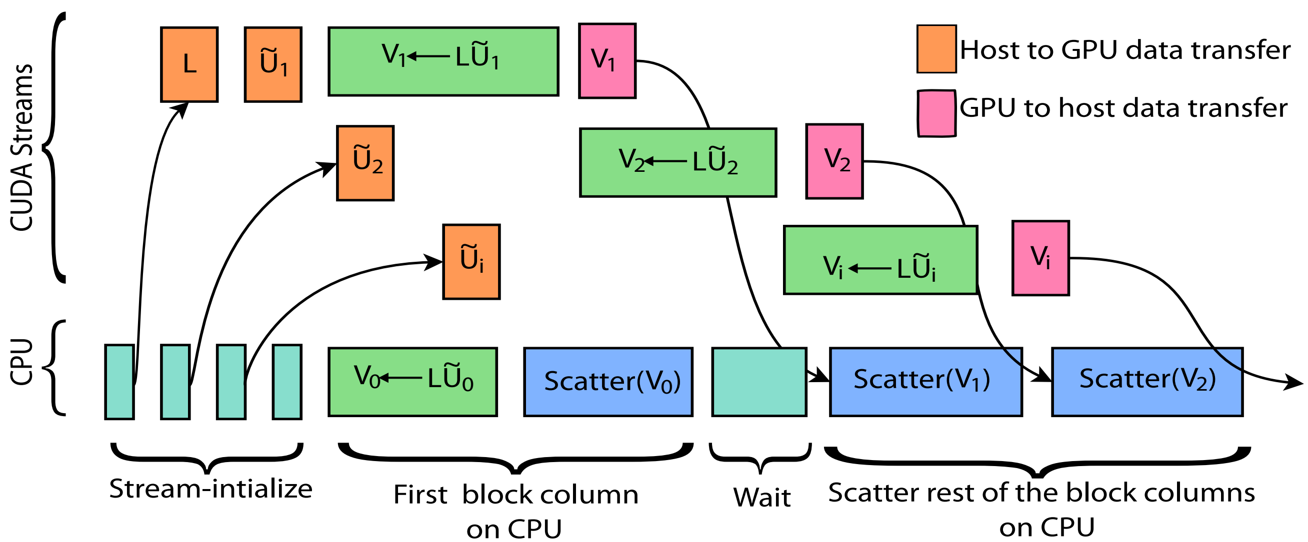
Multicore / GPU-Aware

- New hybrid programming code: MPI+OpenMP+CUDA, able to use all the CPUs and GPUs on manycore computers.
- Algorithmic changes:
  - Aggregate small BLAS operations into larger ones.
  - CPU multithreading Scatter/Gather operations.
  - Hide long-latency operations.
- Results: using 100 nodes GPU clusters, up to 2.7x faster, 2x-5x memory saving.
- New SuperLU_DIST 4.0 release, August 2014.

CPU + GPU algorithm



① *Aggregate small blocks*
② *GEMM of large blocks*
③ *Scatter*



GPU acceleration:
Software pipelining to overlap GPU execution with CPU Scatter, data transfer.

ILU Interface

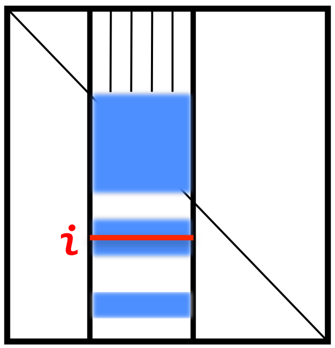
- Available in serial SuperLU 4.0, June 2009
- Similar to ILUTP [Saad]: “T” = threshold, “P” = pivoting
  - among the most sophisticated, more robust than structure-based dropping (e.g., level-of-fill)
- ILU driver: SRC/dgsisx.c

  ILU factorization routine: SRC/dgsitrf.c

  GMRES driver: EXAMPLE/ditersol.c
- Parameters:
  - ilu_set_default_options (&options)
    - options.ILU_DropTol – numerical threshold (τ)
    - options.ILU_FillFactor – bound on the fill-ratio (γ)

Result of Supernodal ILU (S-ILU)

- **New dropping rules S-ILU(τ , γ)**
  - **supernode-based thresholding (τ)**
  - **adaptive strategy to meet user-desired fill-ratio upper bound (γ)**



- **Performance of S-ILU**
  - **For 232 test matrices, S-ILU + GMRES converges with 138 cases (~60% success rate)**
  - **S-ILU + GMRES is 1.6x faster than scalar ILU + GMRES**

S-ILU for extended MHD (fusion energy sim.)

- AMD Opteron 2.4 GHz (Cray XT5)
- ILU parameters: $\tau = 10^{-4}$, $\Upsilon = 10$
- Up to 9x smaller fill ratio, and 10x faster

Problems	order	Nonzeros (millions)	SuperLU Time	fill-ratio	S-ILU time	fill-ratio	GMRES Time	Iters
matrix31	17,298	2.7 m	33.3	13.1	8.2	2.7	0.6	9
matrix41	30,258	4.7 m	111.1	17.5	18.6	2.9	1.4	11
matrix61	66,978	10.6 m	612.5	26.3	54.3	3.0	7.3	20
matrix121	263,538	42.5 m	x	x	145.2	1.7	47.8	45
matrix181	589,698	95.2 m	x	x	415.0	1.7	716.0	289

Tips for Debugging Performance

- Check sparsity ordering
- Diagonal pivoting is preferable
  - E.g., matrix is diagonally dominant, . . .

- Need good BLAS library (vendor, ATLAS, GOTO, . . .)
  - May need adjust block size for each architecture
    (Parameters modifiable in routine sp_ienv())
    - Larger blocks better for uniprocessor
    - Smaller blocks better for parallellism and load balance

  - Open problem: automatic tuning for block size?

Summary

- Sparse LU, ILU are important kernels for science and engineering applications, used in practice on a regular basis
- Performance more sensitive to latency than dense case
- Continuing developments funded by DOE SciDAC projects
  - Integrate into more applications
  - Hybrid model of parallelism for multicore/vector nodes, differentiate intra-node and inter-node parallelism
    - Hybrid programming models, hybrid algorithms
  - Parallel HSS precondtioners
  - Parallel hybrid direct-iterative solver based on domain decomposition

Exercises of SuperLU_DIST

- https://redmine.scorec.rpi.edu/anonsvn/fastmath/docs/ATPESC_2014/Exercises/superlu/README.html

- On vesta:

/gpfs/vesta-fs0/projects/FASTMath/ATPESC-2014/examples/superlu

/gpfs/vesta-fs0/projects/FASTMath/ATPESC-2014/install/superlu

- http://crd.lbl.gov/~xiaoye/SuperLU/slu_hands_on.html

Examples in EXAMPLE/

- **pddrive.c: Solve one linear system**
- **pddrive1.c: Solve the systems with same A but different right-hand side at different times**
  - **Reuse the factored form of A**
- **pddrive2.c: Solve the systems with the same pattern as A**
  - **Reuse the sparsity ordering**
- **pddrive3.c: Solve the systems with the same sparsity pattern and similar values**
  - **Reuse the sparsity ordering and symbolic factorization**
- **pddrive4.c: Divide the processes into two subgroups (two grids) such that each subgroup solves a linear system independently from the other.**

SuperLU_DIST Example Program

- **EXAMPLE/pddrive.c**

- **Five basic steps**
  1. **Initialize the MPI environment and SuperLU process grid**
  2. **Set up the input matrices A and B**
  3. **Set the options argument (can modify the default)**
  4. **Call SuperLU routine PDGSSVX**
  5. **Release the process grid, deallocate memory, and terminate the MPI environment**

Fortran 90 Interface in FORTRAN/

- All SuperLU objects (e.g., LU structure) are **opaque** for F90
  - They are allocated, deallocated and operated in the C side and not directly accessible from Fortran side.
- C objects are accessed via **handles** that exist in Fortran's user space
- In Fortran, all handles are of type INTEGER
- Example: FORTRAN/**f_5x5.f90**

$$A = \begin{bmatrix} s & & u & u & \\ l & u & & & \\ & l & p & & \\ & & & e & u \\ l & l & & & r \end{bmatrix}, \quad s = 19.0,\ u = 21.0,\ p = 16.0,\ e = 5.0,\ r = 18.0,\ l = 12.0$$

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